Combining inclusive and exclusive data analyses-what have we learned so far?

1973 J. Phys. A: Math. Nucl. Gen. 6 733

(http://iopscience.iop.org/0301-0015/6/6/003)

J. Phys. A: Math., Nucl. Gen., Vol. 6, June 1973. Printed in Great Britain. © 1973

Combining inclusive and exclusive data analyses—what have we learned so far?

W Kittel
CERN, 1211 Geneva 23, Switzerland

Received 3 November 1972†

Abstract. A brief summary is given of the main conclusions drawn so far from the model independent study of multiparticle production. A systematic analysis of the data leads to the observation of strong correlations, probably due to three different regimes in particle production: diffraction dissociation, central particle production and nonvacuum exchange processes.

1. What is it all about?

To start historically, I would like to recall that, for the last ten years we have the mini-fashion, for the last five years the maxi-fashion and for the last three years multiparticle production. Up to three years ago, multiparticle production was not a fashion, but mainly the domain of Poles (Polish Poles, in this case, because Regge Poles have been brought in later—by the fashion!).

Since then, many people have joined in and one can group them into (*a*) those who have joined *in spite* of there being no theory and (*b*) those who have joined *because* there is no theory. The review papers are counted in kilograms already, so that (not to add too much weight) there remains nothing but to try to ask: (i) what should we learn (eventually)? and (ii) what have we learned (so far)?

The answer to the first question is obvious, even though it sounds still very ambitious: We should learn how to write a closed theory of strong interactions.

The answer to the second question is much less obvious and ambitious, but before we try to formulate it we shall start with what it is all about.

A typical multiparticle reaction of particle A with particle B giving n particles C_i in the final state (reaction channel of multiplicity n) can be written

$$\mathrm{A}+\mathrm{B} \rightarrow \mathrm{C}_1+\ldots+\mathrm{C}_l+\mathrm{C}_{l+1}+\ldots+\mathrm{C}_n, \tag{1}$$

where particles $C_1, \ldots C_l$ are 'observed' and particles $C_{l+1}, \ldots C_n$ are not (Van Hove 1971). It is quite generally accepted now, that reactions of this type have to be studied, because: (i) a theory of strong interactions should be able to describe them as well as two-particle final states; (ii) they are responsible for about 75% of the total cross section; (iii) they are connected to the remaining 25% via unitarity; and (iv) they are, as we shall see, very rich in themselves (and thus a possible source of knowledge).

It is also quite generally accepted that reactions of type (1) are very difficult to study since they have to be studied in phase space and phase space is $3n$–4 dimensional.

† Review talk given at The Institute of Physics Conference on Elementary Particle Physics, Southampton, 12–14 September 1972.

Therefore, the first step is to invent simplifications, that is, reductions of the dimensionality of the analysis giving differential information comprehensively. Two of these simplifications are the 'inclusive' and the 'exclusive' data analyses.

In the 'inclusive' data analysis (Feynman 1969 and Benecke *et al* 1969) one keeps l fixed and (sometimes too) small and sums over all n in reaction (1). In the 'exclusive' data analysis one keeps $n = l$ fixed and studies an individual reaction channel where the nature and the momenta of all final state particles are known (Van Hove 1969a, b, Bialas *et al* 1969 and Kittel *et al* 1971). Both methods have been invented to be used at 'high energies'.

Indeed, great help comes from our knowledge of cosmic ray (and now ISR) experiments in the TeV (10^{12} eV) region, which tell us that: (i) the average multiplicity $\langle n\rangle$ increases much slower with available energy than allowed kinematically; (ii) the transverse variables are, in first order, particle independent, limited to small values and largely decoupled from the longitudinal ones.

While knowledge of the first point gives us some necessary optimism that the whole dimensionality problem will not explode immediately, the second point suggests a possible way to reduce the dimension of our investigation space from $3n$–4 to n–2 by using longitudinal phase space (LPS) only (Van Hove 1969a, b). As we shall see, this method can be applied equally well for inclusive and exclusive data analyses. Transverse momenta which are less energy, multiplicity and particle dependent than longitudinal momenta (and apparently limited by dynamics to have an average of 0·3–0·5 GeV/c only), can easily be introduced later and treated in the second approximation (Bialas *et al* 1969, Brau *et al* 1971). While this second approximation is of great help in a detailed study of special questions, we shall here discuss the results in terms of longitudinal variables only.

2. Inclusive single-particle distributions

Total cross sections, plotted in figure 1 against the incoming laboratory momentum (Denisov *et al* 1971, Giacomelli 1972) are otherwise completely integrated quantities and can therefore be regarded as inclusive distributions of order zero. One step further

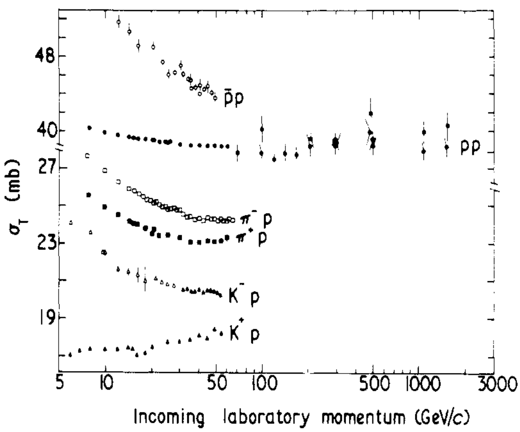

Figure 1. Total cross sections against the incoming laboratory momentum (Denisov *et al* 1971, Giacomelli 1972).

towards a differential data analysis are Lorentz-invariant inclusive single-particle distributions

$$f_1(s, p_{\parallel}, p_{\perp}) = \frac{E}{\pi} \frac{\mathrm{d}\sigma_1}{\mathrm{d}p_{\parallel}\, \mathrm{d}p_{\perp}^2} \tag{2}$$

of particle C with longitudinal momentum $p_{\parallel}$, transverse momentum $p_{\perp}$ and energy E produced in the reaction

$$\mathrm{A}+\mathrm{B} \rightarrow \mathrm{C}+\text{anything} \tag{3}$$

at the total CMS energy $s^{1/2}$.

For several reasons it has proved useful to replace $\mathrm{d}p_{\parallel}/E$ in equation (2) by $\mathrm{d}y$, where y is the longitudinal rapidity (Feynman 1969) of particle C,

$$y = \ln \frac{E+p_{\parallel}}{m_{\parallel}}, \tag{4}$$

defined in the same reference frame as E and $p_{\parallel}$, with $m_{\parallel}^2 = m^2 + p_{\perp}^2$.

The distribution

$$g_1(s, y, p_{\perp}) = \frac{1}{\pi} \frac{\mathrm{d}\sigma_1}{\mathrm{d}y\, \mathrm{d}p_{\perp}^2} \tag{5}$$

for the π^- and π^+ produced in

$$pp \rightarrow \pi^- + \text{anything} \tag{6}$$

and

$$pp \rightarrow \pi^+ + \text{anything} \tag{7}$$

is given in figure 2 by Sens (1972) in the rest frame of one of the incoming protons, for three different transverse momenta and for incoming laboratory momenta $p_{\mathrm{in}}^{\mathrm{lab}}$ between 12 and 1500 GeV/c. We notice that, except for small y^{lab}, the differential cross section increases with $p_{\mathrm{in}}^{\mathrm{lab}}$ and this very similarly for all three transverse momenta shown.

As pointed out in § 1, one can integrate over the transverse momenta without loosing too much information and study the distribution

$$G_1(s, y) = \int g_1(s, y, p_{\perp})\, \mathrm{d}^2\boldsymbol{p}_{\perp} \tag{8}$$

in longitudinal phase space. This is shown for y^{lab} in figure 3(a) with the sum of reactions (6) and (7), again for $p_{\mathrm{in}}^{\mathrm{lab}}$ between 12 and 1500 GeV/c (Breidenbach *et al* 1972, Mück *et al* 1972, Franz 1972, private communication).

The curves are to guide the eye† and are drawn with a point of symmetry corresponding to a longitudinal CMS rapidity $y^* = 0$.

The shape of the y^{lab} distribution remains invariant under longitudinal Lorentz transformations. So, one can easily redraw the distributions of figure 3(a) in the CMS—This has been done in figure 3(b).

As one can see from figure 3, the y distribution has a flat plateau developing at ISR energies, tailing off on the two sides. At around 20 GeV/c, this plateau has not been reached yet, so that the whole distribution seems to consist of two tails only.

† For a more complete compilation we refer to the Proceedings of the 16th International Conference on High Energy Physics, Batavia 1972.

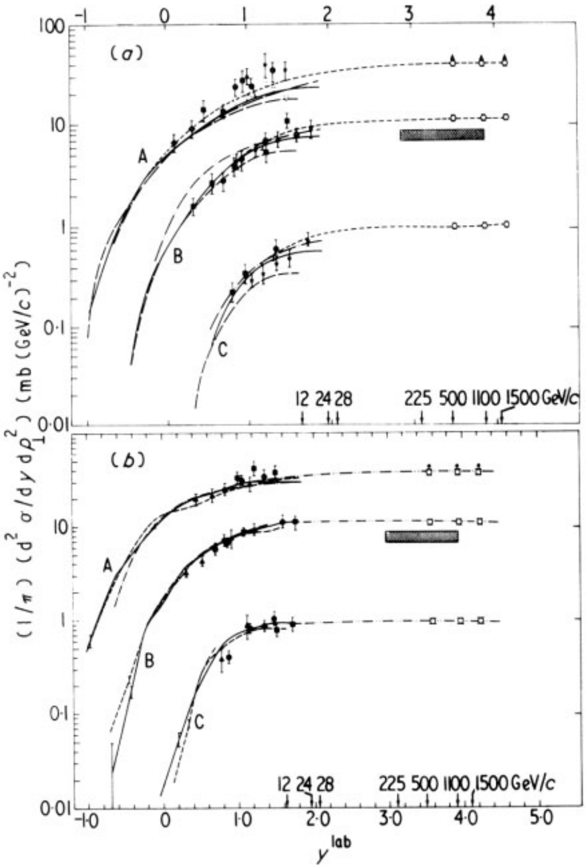

Figure 2. Inclusive single-particle π^- and π^+ distribution for pp interactions against y^{lab} at the incoming laboratory momentum indicated. The arrows indicate the position of the symmetry line at CMS rapidity $y^* = 0$. (*a*) pp $\rightarrow \pi^-$ +anything; (*b*) pp $\rightarrow \pi^+$ +anything; curves A, $p_\perp = 0{\cdot}2$ GeV/c; curves B, $p_\perp = 0{\cdot}4$ GeV/c; curves C, $p_\perp = 0{\cdot}8$ GeV/c. Data points and curves are the same as in (Sens 1972).

The overlap of the two tails is indeed clearly visible in charge distributions suggested by Van Hove (1971). They are shown in figure 4 for $\pi^\pm$p reactions at 16 GeV/c by the ABBCCHW collaboration (Morrison 1972a), as functions of the reduced longitudinal rapidity (Van Hove 1971),

$$\xi = \frac{y - y_{\mathrm{B}}}{y_{\mathrm{A}} - y_{\mathrm{B}}} \tag{9}$$

and the reduced CMS longitudinal momentum (Feynman 1969),

$$x = \frac{p_{\parallel}^*}{p_{\mathrm{in}}^*}. \tag{10}$$

Before the collision, all charge is accumulated at $\xi = 0\,(x = -1)$ and $\xi = 1\,(x = +1)$. After the collision, one would expect the charge to spread because of dissociation or fragmentation of the incident particles, but to vanish in the centre. At 16 GeV/c the contrary is true. The dQ/dξ distribution for π^+p reactions increases suprisingly smoothly

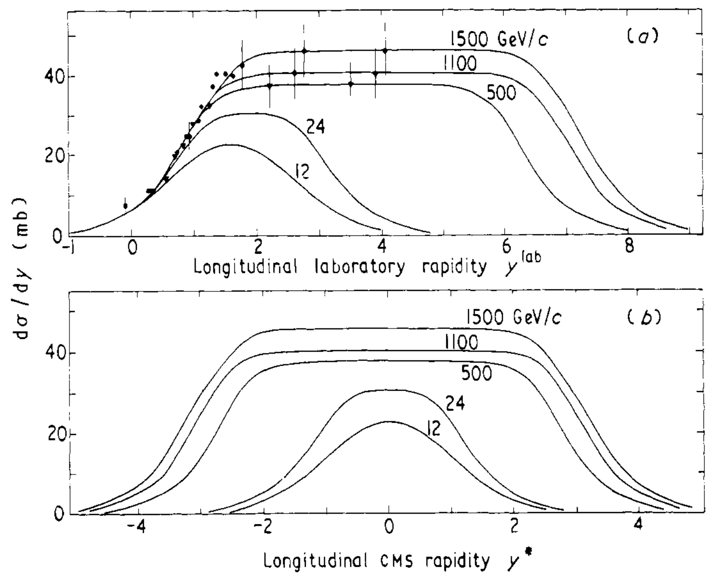

Figure 3. Inclusive single-pion distribution for pp interactions between 12 and 1500 GeV/*c* integrated over the transverse momenta, and plotted against the longitudinal rapidity in the proton rest system, y^{lab} (figure 3(*a*)) (Breidenbach *et al* 1972, Mück *et al* 1972, Franz 1972), and in the CMS, y^* (figure 3(*b*)).

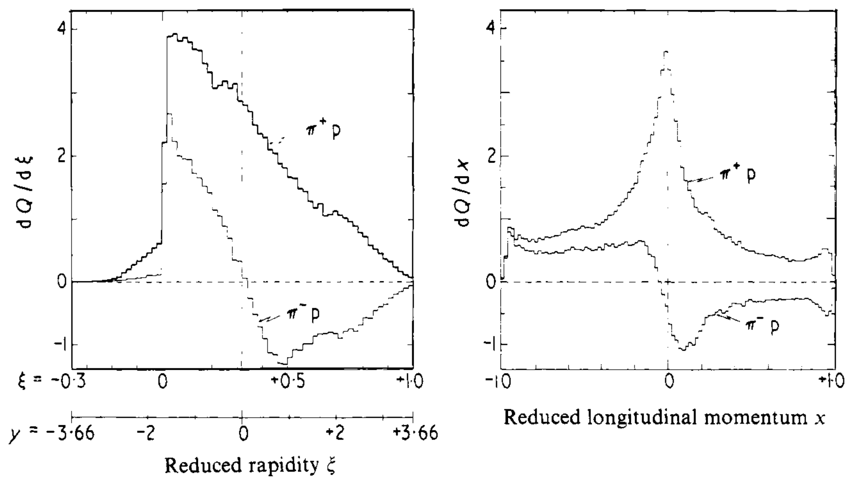

Figure 4. Distribution of charge in (*a*) the reduced longitudinal rapidity ξ and (*b*) the reduced longitudinal momentum x for π^+p and π^-p inelastic reactions at 16 GeV/*c* (Morrison 1972a).

when going from $\xi = +1$ to $\xi = 0$. For π^-p it is forced to pass through $dQ/d\xi = 0$ near the centre, but the maximum negative charge is much closer to the centre than to $\xi = 1$. The dQ/dx distributions (figure 4(*b*)) exhibit some residual charge maxima near $x = \pm 1$, but the main feature is the central peak for π^+p reactions and the overlap of a positive and a negative peak near $x = 0$ for π^-p reactions.

The central peaking effect is, of course, partially due to the large cross section at $x = 0$ as in figures 5(a, b) for π^-p and π^+p induced pion production, respectively. Therefore, the corresponding charge ratio is plotted in figures 5(c, d) as a function of x. It should be unity for pair production, if this is independent of the incoming charge. In figure 5 one can see that the charge ratio is not unity at $x = 0$, and even comes closest to unity at a nonzero x value.

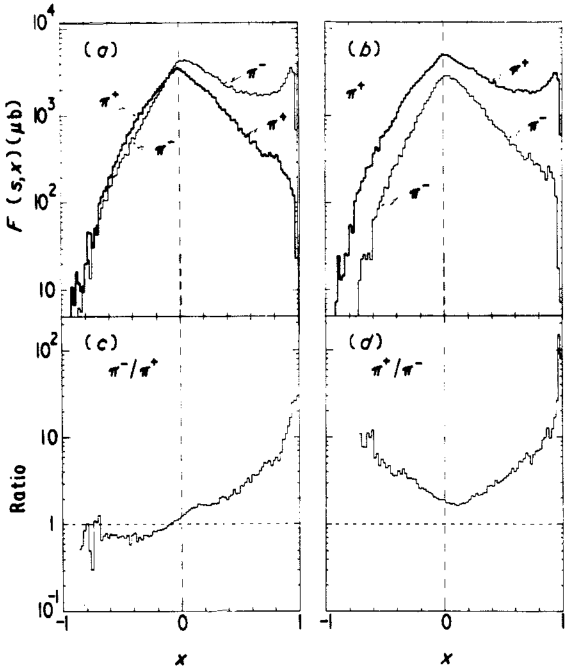

Figure 5. Inclusive single-particle π^+ and π^- distribution for (a) π^-p and (b) π^+p reactions at 16 GeV/c. Parts (c) and (d) are charge ratios from the above pion distributions (Morrison 1972a).

As shown by Morrison (1972) in figure 6, the charge ratio at $x = y^* = 0$ (corresponding to a CMS production angle $\theta^* = 90°$) approaches unity rather quickly ($s = \infty$ at the right edge in this scale), probably well below ISR energies. This seems to hold for the very central x region only, however, since at $x = 0{\cdot}2$ the π^+/π^- ratio is about 2, even at ISR energies (Bertin *et al* 1972).

So, only at $s \to \infty$, central ($\pi^+\pi^-$) production may become energy independent (scaling) and independent of the incident particles (factorization). The latter is shown once more for the general case of arbitrary incoming particles in a compilation by Ferbel (1972) in figure 7 ($s = \infty$ on the left edge in this scale), Meyer and Struczinski (1972) and Idschok *et al* (1972). Still more generally, this may eventually become true for arbitrary particle–antiparticle pair production.

On the other hand, energy independence (scaling) and factorization are known to hold reasonably well in the region of the tails of the y distribution, in the case of early limiting (Chan Hong-Mo *et al* 1971) even in the 30 GeV/c region (see eg figures 2 and 3(a) for scaling and figure 8 for factorization (Moffeit *et al* 1972)).

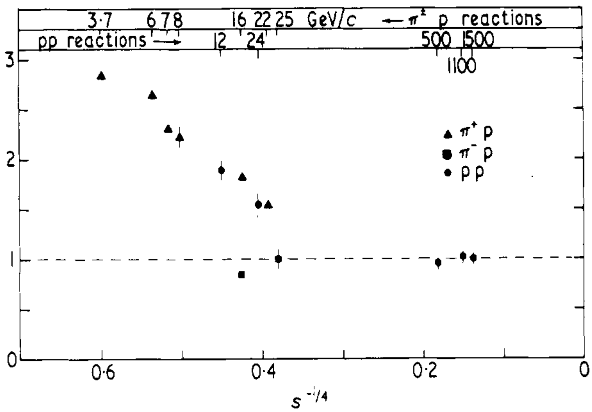

Figure 6. Ratio of π^+ and π^- production at $y^* = x = 0$ in π^+p, π^-p and pp reactions, as a function of $s^{-1/4}$ (Morrison 1972a).

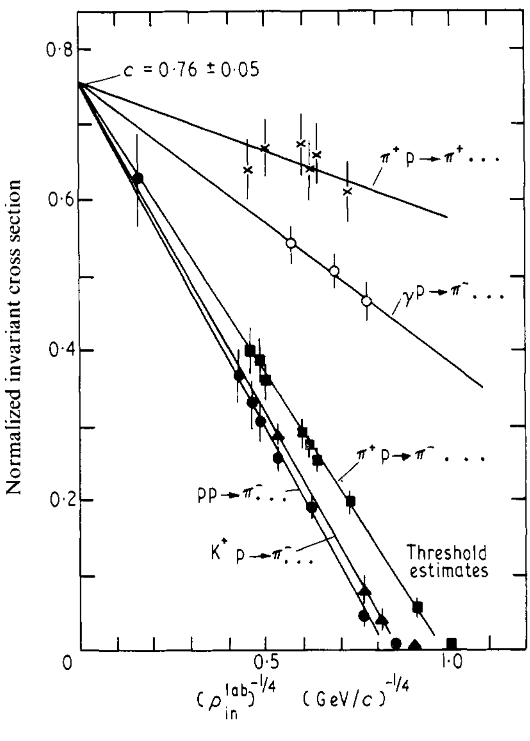

Figure 7. Inclusive single-particle cross section at $x = 0$ as a function of the incoming laboratory momentum, normalized by the total cross section at $s = \infty$ (Ferbel 1972).

So we have learned that we have to deal with essentially two regions: one where at least one incident particle breaks up, independent of the energy and nature of the other particle; and one region where particles are produced on the average in charge-pairs eventually independent of the energy and of the incident particles. The overlap of these two regions seems to be very large and smooth, especially in x but also in y

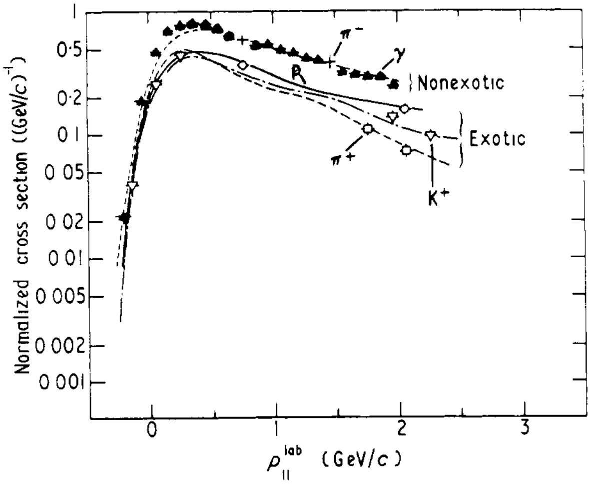

Figure 8. Inclusive π^- distribution in the longitudinal laboratory momentum $p_{\parallel}^{lab}$ for pp, K^+p, π^+p, π^-p and γp reactions normalized by the total cross section at $s = \infty$ (Moffeit *et al* 1972).

distributions, and depends on the energy. Coming back to our examples in the introduction, inclusive single-particle distributions can in more than just one aspect be best compared to the maxi-fashion. They may be elegant, but....

3. Correlations in inclusive distributions

A lot more detail is expected to be seen in inclusive two-particle distributions. First because they are more differential than single-particle distributions, but mainly because they allow a study of correlations in particle production. Correlations, in turn, are expected to give additional information on the production mechanism.

Similarly to equation (2), the Lorentz-invariant distribution of particles C_1 and C_2, produced in the inclusive reaction

$$A + B \rightarrow C_1 + C_2 + \text{anything}, \tag{11}$$

can be written as

$$f_2(s, \boldsymbol{p}_1, \boldsymbol{p}_2) = E_1 E_2 \frac{d^6\sigma_2}{dp_{\parallel 1}\, dp_{\parallel 2}\, d^2\boldsymbol{p}_{\perp 1}\, d^2\boldsymbol{p}_{\perp 2}}. \tag{12}$$

As in equation (5), $dp_{\parallel i}/E_i$ can, of course, be replaced by dy_i. Even though some information can be obtained from the correlations in the three independent transverse variables, one can again integrate over these and concentrate on the correlations in the integrated function, for example, in

$$F_2(s, x_1, x_2) = \int f_2(s, x_1, x_2, \boldsymbol{p}_{\perp 1}, \boldsymbol{p}_{\perp 2})\, d^2\boldsymbol{p}_{\perp 1}\, d^2\boldsymbol{p}_{\perp 2}. \tag{13}$$

Correlations of various order have been studied in some detail, mainly following the pattern of the cluster expansion in statistical mechanics (for a summary see Koba 1972 and Morrison 1972b). Because of the relatively small number of particles involved and the close vicinity of nonsquare 'walls' due to the phase space boundary, we abandon this method for the moment and study correlations in the shape of the two-particle distribution directly. This has the additional advantage that no information is lost because of subtractions or divisions of distribution functions (Van Hove 1971).

The ABBCCHW collaboration (1972) has studied correlations between the proton and the most forward (fastest) 'beam-like' pion in the reactions

$$\pi^+ p \rightarrow \pi_f^+ + p + \text{anything} \tag{14}$$

at 8 and 16 GeV/c and

$$\pi^- p \rightarrow \pi_f^- + p + \text{anything} \tag{15}$$

at 16 GeV/c. In figure 9 is given the 'inclusive' two-particle distribution in the reduced longitudinal momenta x_f and x_p for the fastest pion and the proton of reaction (14), respectively, at the two energies. Each of the little bars represents the cross section $F_2(s, x_f, x_p)$ integrated over the corresponding two-dimensional (x_f, x_p) bin.

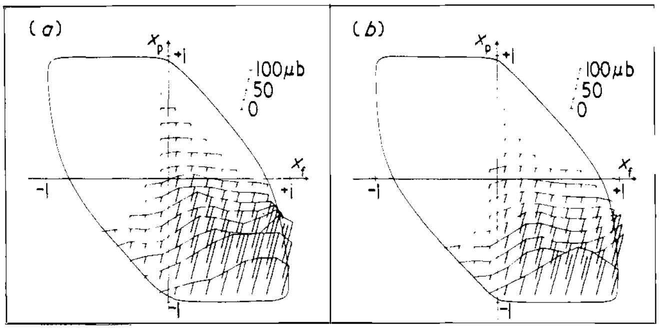

Figure 9. Inclusive two-particle distribution $(p_{in}^*)^{-2} E_f^* E_p^* \, d\sigma/dx_f \, dx_p$ for the reaction $\pi^+ p \rightarrow \pi_f^+ + p + \text{anything}$ at (a) 8 and (b) 16 GeV/c (ABBCCHW collaboration 1972).

The fact that almost all events lie in the sector on the lower right can already be considered as correlation. This is a trivial one however and comes from our selection of one preferentially backwards and one preferentially forwards produced particle. Further correlations are obviously present: for $x_p \simeq 0$, the x_f distribution has one single, rather flat maximum for small x_f. For $-\frac{1}{8} > x_p > -\frac{7}{8}$, one at small x_f and one for large x_f. Finally, for $x_p \simeq -1$ the x_f distribution shows again only one maximum but this time at $x_f \simeq 0{\cdot}6$. This is certainly a peculiar shape, not obvious from single-particle distributions. In particular the 'valley' parallel to the x_p axis around $x_f = 0{\cdot}6$ would be hidden by the 'peak' it ends in at large negative x_p values. Already this effect justifies further study.

The energy dependence, parametrized as

$$F_2(s, x_f, x_p) \propto (p_{in}^{lab})^{-N(x_f, x_p)}, \tag{16}$$

is given in figure 10, where $N(x_f, x_p)$ is plotted against x_f and x_p, positive N meaning decreasing cross section. Shading the regions where the cross section is increasing or only slowly decreasing ($N < 0{\cdot}5$), we observe that this is indeed the case just for the two maxima along the phase space boundary and that for small x_f and medium negative

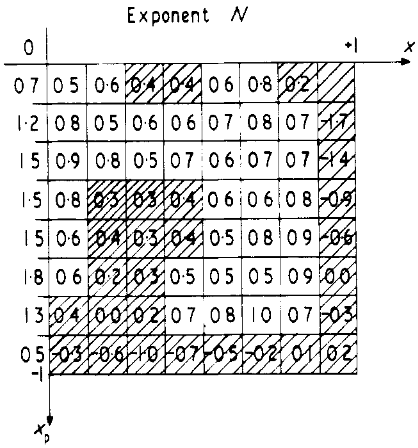

Figure 10. Exponent $N(x_f, x_p)$, parametrizing the energy dependence of the two-particle distribution $\pi^+p \rightarrow \pi_f^+ + p + \text{anything}$ between 8 and 16 GeV/c. The statistical errors are of order 0·1 (ABBCCHW collaboration 1972).

x_p. So we can expect that the three maxima may become even more clearly separated at higher energies. We shall see in the next section that this is probably due to the decrease of the cross section for the exchange of quantum numbers, in particular $\Delta^{++}\rho^0$ production.

The distribution for the π^-p reaction (15) shows structure very similar to that of the π^+p reaction (14) at the same energy. The inclusive π_f^- against p distribution is decomposed as far as possible into the individual channels in figure 11. Immediately we see that there are strong correlations in the two x variables also in the exclusive distributions, especially for low multiplicities: low multiplicity channels have a high cross section close to the phase space boundary, as expected for the kinematical configuration of diffraction dissociation or fragmentation of one of the incoming particles. High multiplicity channels, on the other hand, give rise to population of more central regions, in particular of the third maximum. This trend is so strong that high multiplicities can be separated quite well from low ones by simply cutting in x_p and x_f.

Coming back to the inclusive distribution of figure 9, the correlation in the sum is of course not equal to the sum of correlations. However, in our special case of three maxima and a distinct 'valley', a lot of the correlation in the sum can be understood as being due to an interplay of multiplicity $n \leqslant 4$ with $n \geqslant 6$.

So we have learned, that there exists strong correlation in the p against $\pi_f^{\pm}$ longitudinal distribution, due to the separation of low multiplicity fragmentation from some mechanism leading to high multiplicity collisions with more central particle production. Figure 11 also leads us to the next two sections, where we further study the correlations in individual exclusive channels on one hand and the 'conspiracy' between the different exclusive channels on the other.

4. Correlations in exclusive distributions

Keeping the multiplicity n reasonably small allows us to study reactions of the type

$$A + B \rightarrow C_1 + C_2 + \dots C_n \tag{17}$$

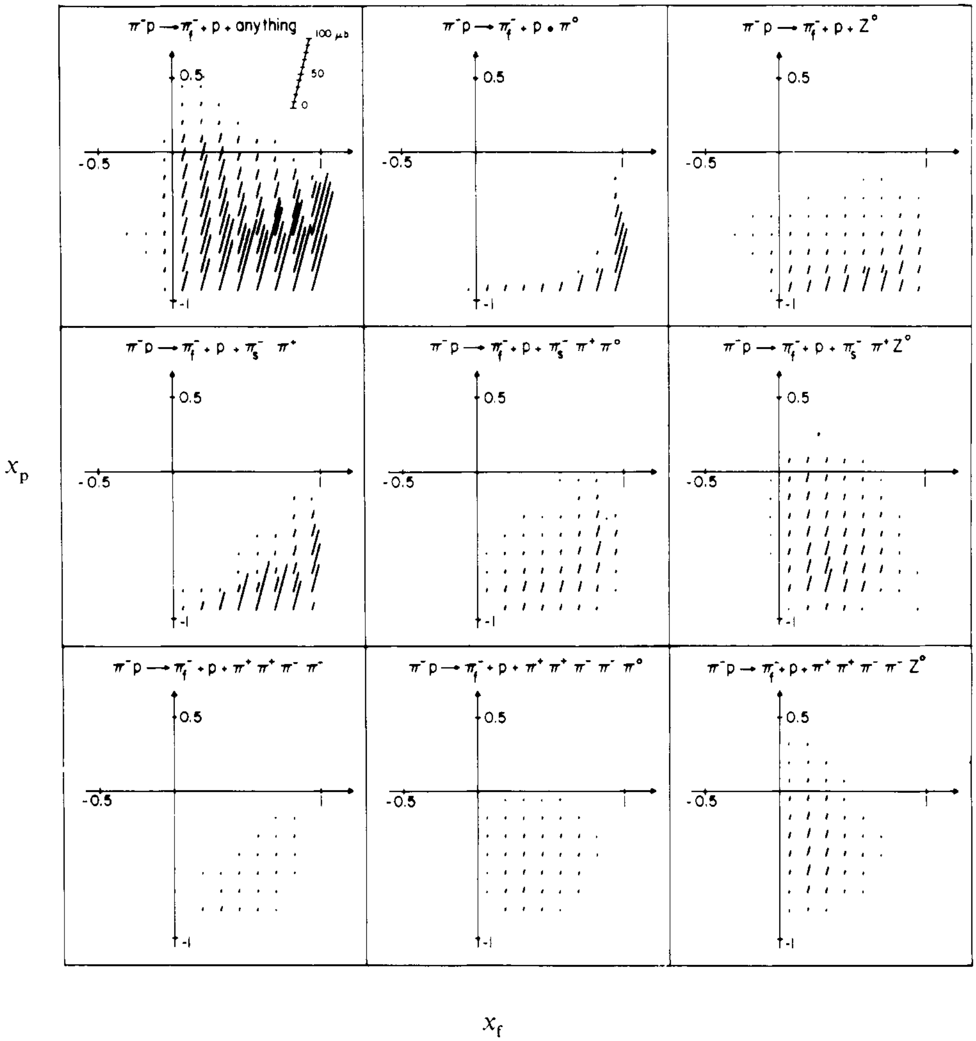

Figure 11. Decomposition of the inclusive two-particle distribution for $\pi^- p \to \pi_f^- + p +$ anything at 16 GeV/c into the indicated channels, Z^0 standing for two or more neutral pions (ABBCCHW collaboration 1972).

'exclusively', when all (or all but one) particles are observed and their momenta are known. In such a reaction, the number of independent longitudinal momenta is reduced from n to $n-2$ by the longitudinal momentum and energy constraints. A plot of the type shown in figure 11 therefore fully parametrizes a four-particle final state in longitudinal phase space, so we shall deal here mainly with this multiplicity. The analysis can easily be generalized to other multiplicities and has been used successfully for $3 \leqslant n \leqslant 8$ (so called 'few-body' reactions).

Figure 12(a) gives a longitudinal phase space plot for the four-body reaction

$$\pi^- p \to \pi^- \pi^- \pi^+ p \tag{18}$$

at 11 GeV/c and 16 GeV/c (Kittel *et al* 1971), where each of the little bars corresponds

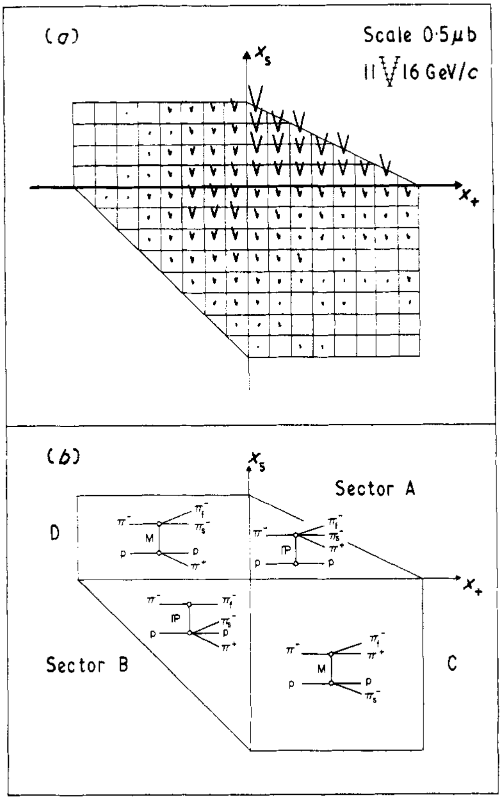

Figure 12. (a) weighted LPS distribution for $\pi^-p \to 2\pi^-\pi^+p$ at 11 and 16 GeV/c and (b) diagrams representing the exchange mechanism expected to dominate the corresponding sector at high energies (Kittel *et al* 1971).

to the cross section in the two-dimensional (X_1, X_2) bins, corrected for the phase space density (ie to the squared matrix element divided by s^2 and integrated over the transverse momenta, $s^{-2}\overline{|M|^2}$). The reduced longitudinal momenta X_i are defined for exclusive reactions as

$$X_i = \frac{p^*_{i\parallel}}{\frac{1}{2}\Sigma|p^*_{i\parallel}|}. \tag{19}$$

The advantage of the X_i is the fact that their two constraints

$$\sum_i^n X_i = 0, \qquad \sum_1^n |X_i| = 2 \tag{20}$$

are independent of transverse momenta and of the total energy. While differing from the Feynman variables x_i defined in equation (10) at small energies, they become identical to these as $s = \infty$.

The most differential distribution is obtained when using the reduced longitudinal momenta X_+ for the positive pion in reaction (18) and X_s for the slower (less forward produced) negative pion. These particles turn out to be dynamically allowed to cover the full longitudinal phase space (compare figures 11 and 12). Using these variables in figure 12, we immediately see strong correlation, even clearer than in figure 11.

In particular, the cross section is much larger in the two triangular sectors than in the two squares.

'Kinematical' diagrams, expected to achieve some dynamical meaning at 'high energies', are given for the four sectors of the plot in figure 12(*b*). As far as quantum numbers are concerned, vacuum (pomeron) exchange P is allowed in the triangular sectors and charged meson exchange M with G parity $G = -1$ in the rectangles. Multiperipheral processes, on the other hand, are expected to contribute near the sector boundaries. However, in figure 12(*a*), the cross section near the sector boundaries looks rather like a tail from the adjacent sectors, so that large contributions from multiperipheral mechanisms do not seem necessary to explain the shape of the distribution.

Figure 13 gives the exponent N defined in equation (16) as a function of X_s and X_- for the reaction

$$\pi^+ p \rightarrow \pi_f^+ \pi_s^+ \pi^- p \tag{21}$$

between 8 and 16 GeV/c (Beaupré *et al* 1972a). A similar figure exists for the reaction

$$K^+ p \rightarrow K^+ \pi^+ \pi^- p \tag{22}$$

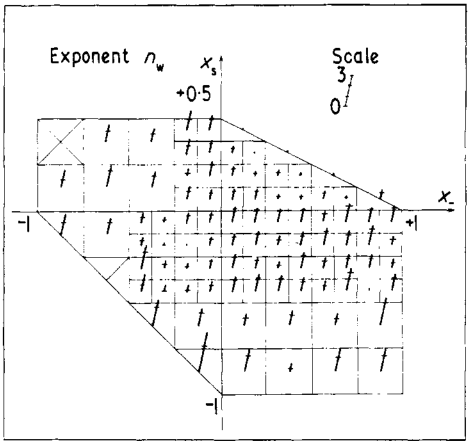

Figure 13. Exponent $N(X_-, X_f)$, parametrizing the energy dependence of the weighted LPS distribution for $\pi^+ p \rightarrow \pi_f^+ \pi_s^+ \pi^- p$ between 8 and 16 GeV/c (Beaupré *et al* 1972a).

at several energies between 5 and 12·7 GeV/c (De Wolf *et al* 1972). For both cases, the exponent is in general much smaller than unity over the meson-dissociation sector A and in the centre of the proton-dissociation sector B, while it is $N \simeq 2$ in the charge exchange sectors C and D. In sectors A and B we should find energy independence (scaling) and factorization as for the tails of the single particle inclusive distribution in § 2, but we should of course not expect diffraction dissociation to be 'cleaner' and less energy dependent than elastic scattering.

Thus, at high enough energies, the sector boundaries separate particles produced as a forward system from those produced as a backward system. At this stage, the LPS separation becomes a simplification of the 'principal axis' analysis of multiparticle reactions suggested already in 1964 by Brandt, Peyrou, Sosnowski and Wroblewski (Brandt *et al* 1964). As pointed out by these authors, such a separation allows a detailed study of individual systems ('intermediate bodies') with minimum contamination from spurious particle combinations.

The mass distribution in the corresponding LPS sectors A and B is given in figure 14 for reactions (18) and (21), $\pi^{\pm}p \rightarrow \pi^{\pm}\pi^{+}\pi^{-}p$, at 16 GeV/$c$. Both the shape and the magnitude of the distribution are very similar for $\pi^{+}p$ and $\pi^{-}p$ induced pion and proton dissociation (or fragmentation).

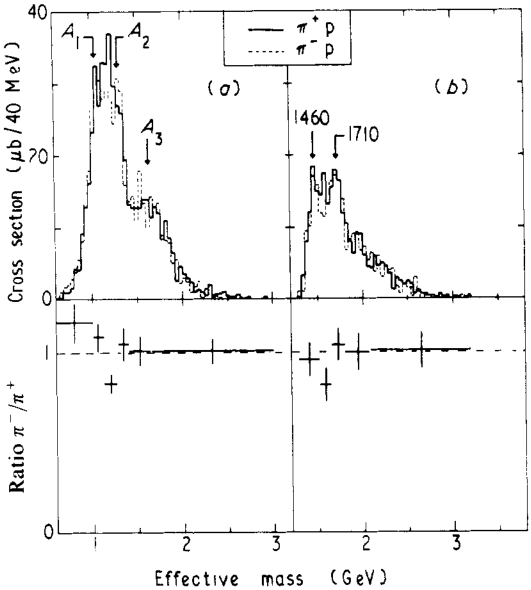

Figure 14. Comparison of (a) ($\pi^{\pm}\pi^{+}\pi^{-}$) and (b) ($p\pi^{+}\pi^{-}$) mass distributions in the pion and proton-dissociation sectors of LPS respectively, for $\pi^{\pm}p \rightarrow \pi^{\pm}\pi^{+}\pi^{-}p$ at 16 GeV/c (Beaupré *et al* 1972b).

Factorization has been shown to hold for the proton vertex by Yamdagni and Ljung (1971) and by the ABCLV collaboration (Beaupré *et al* 1971) as

$$R = \frac{\sigma_{w}(Mp \rightarrow M(p\pi^{+}\pi^{-}))}{\sigma_{w}(pp \rightarrow p(p\pi^{+}\pi^{-}))} = \frac{\sigma_{el}(Mp \rightarrow Mp)}{\sigma_{el}(pp \rightarrow pp)}, \tag{23}$$

where σ_w is the phase space weighted cross section and M stands for a $\pi^{\pm}$ or $K^{\pm}$ meson. The ratio R for $M = \pi^{-}$ is given in figure 15 for the proton-dissociation sector and agrees at least in the centre with the expected value of $R \simeq 0{\cdot}43$ for elastic scattering.

The low mass (3π) system has been studied in great detail by Illinois (Ascoli *et al* 1970) and by the CERN-IHEP collaboration (Antipov *et al* 1972) in terms of a partial-wave decomposition. With the exception of some $J^P = 2^+$ it is essentially all in s-wave $\pi\epsilon(0^-)$, $\pi\rho(1^+)$ and $\pi f(2^-)$, thus in the same spin-parity series as the incoming pion (see figure 16). One also knows about both the ($M\pi\pi$) system and the ($p\pi\pi$) system that helicity is not conserved in the s-channel, nor probably in the t-channel (for a summary see Otter 1972).

As pointed out above, we cannot expect dissociation to proceed by pure vacuum (pomeron) exchange at around 20 GeV/c. We know that elastic scattering has a 'cross-over' in the four-momentum transfer distributions, usually interpreted as interference

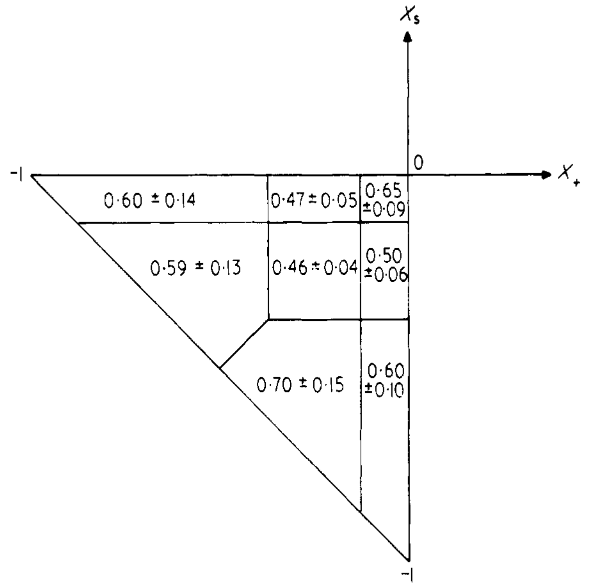

Figure 15. Ratio of the weighted cross sections for $\pi^-p \rightarrow \pi_f^-\pi_s^-\pi^+p$ at 16 GeV/c and $pp \rightarrow p\pi^+\pi^-p$ at 19 GeV/c, as a function of X_+ and X_s (Yamdagni and Ljung 1971).

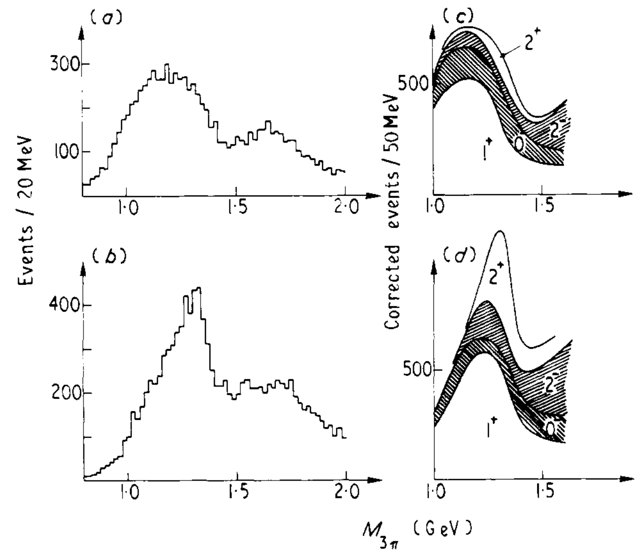

Figure 16. Partial-wave decomposition of the (3π) system produced in $\pi^-p \rightarrow \pi^-\pi^-\pi^+p$ at 40 GeV/c, for two different t intervals: (a) and (c) $0{\cdot}04 < t < 0{\cdot}17\,(\text{GeV}/c)^2$; ($b$) and ($d$) $0{\cdot}17 < t < 0{\cdot}33\,(\text{GeV}/c)^2$ (Antipov *et al* 1972).

between exchanges even and odd under C-conjugation. Indeed a similar cross-over effect (shown in figure 17) is found by SLAC (Brandenburg *et al* 1972) and the ABBCH collaboration (Beaupré *et al* 1972b) for diffractive Q^0, $\overline{Q}^0$ and $(3\pi)^{\pm}$ production, respectively.

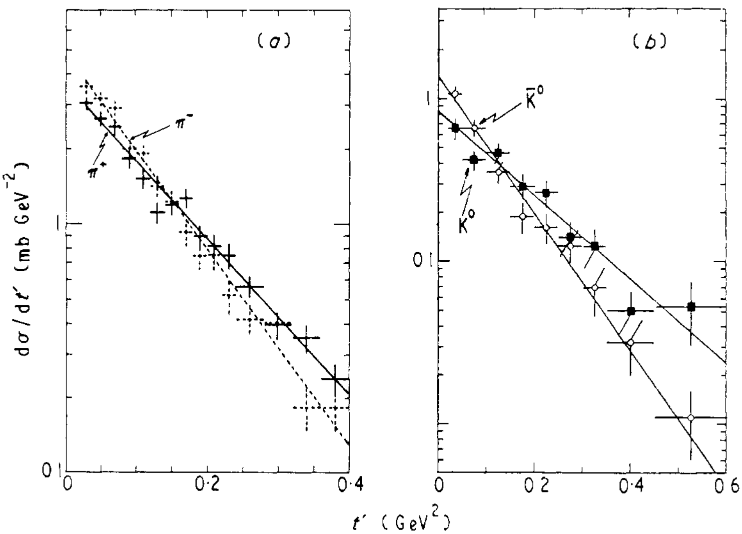

Figure 17. Four-momentum transfer t' distribution for (a) the reactions $\pi^-p \rightarrow \pi^+\pi^-\pi^-p$ and $\pi^-p \rightarrow \pi^-\pi^-\pi^+p$ at 16 GeV/c in the pion dissociation sector of LPS (Beaupré *et al* 1972b) and (b) for the reactions $K^0p \rightarrow Q^0p$ and $\bar{K}^0p \rightarrow \bar{Q}^0p$ between 4 and 12 GeV/c (Brandenburg *et al* 1972).

Very interesting experimental information about competition and interference between vacuum and quantum-number exchanges in proton dissociation has further been obtained by several groups from isospin decomposition of three-particle and quasi-three-particle final states (for a summary we refer to the review talks of Rushbrooke (1972) and Vebeure (1972a, b)).

So our rather detailed knowledge of internal structure in production and decay of two- and three-particle 'clusters' goes far beyond a zero approximation isotropic treatment of such systems.

Sectors C and D contain reactions of completely different type. As pointed out for figure 13, the cross section is definitely decreasing there with increasing energy. Distributions in the proper mass combinations, as those in figure 18 for

$$\pi^+p \rightarrow (\pi_f^+\pi^-)(\pi_s^+p), \tag{24}$$

show that these sectors are populated by double resonance production wherever the two-particle system has nonexotic quantum numbers. Coming back to figures 9–11, it turns out that the decrease of the cross section for the charge exchange reaction (24) with increasing energy is to a large extent responsible for the deepening of the 'valley' between the three distinct peaks in the inclusive two-particle distribution discussed in § 3.

So we have learned that the reason for the strong correlation in four-body reactions are dominant diffraction dissociation or fragmentation with reasonably known internal structure on one hand and exchange processes giving well defined resonances on the other. These processes can be clearly separated in longitudinal phase space, already at about 20 GeV/c.

Two questions remain to be settled. The first one, suggested by the observation of double-resonance production above, is that of double diffraction-dissociation (Kittel *et al* 1971). As reviewed in some detail by Verbeure (1972a, b) and by Rushbrooke

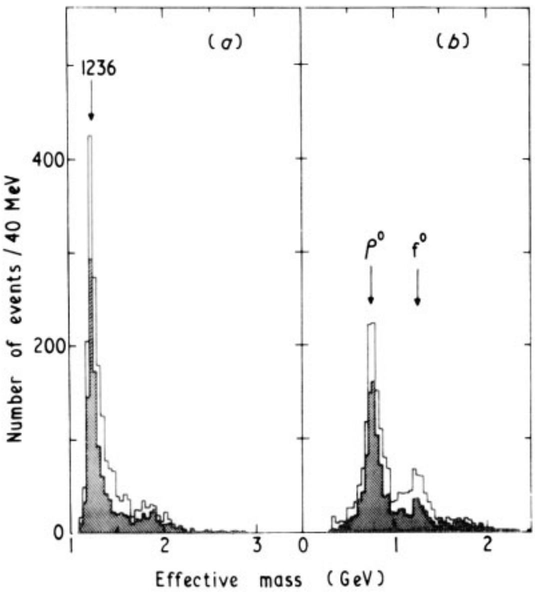

Figure 18. Effective mass distribution of (*a*) the ($p\pi_s^+$) and (*b*) ($\pi_f^+\pi^-$) systems in the charge-exchange sector C for $\pi^+p \to \pi^+\pi^+\pi^-p$ at 16 GeV/*c* (Beaupré *et al* 1972a).

(1972), this process is reasonably well established in five- and six-body reactions including those coming from photoproduction, but is probably not as 'clean' as single diffraction-dissociation. Very interesting work has been done, especially concerning the exchange of quantum numbers between the forward and backward hemispheres in even higher multiplicities (Deutschmann *et al* 1972a, b). Nevertheless we have to wait for higher energies to separate particles coming from meson and proton vertices clearly and possibly to resolve the third peak in the inclusive distribution of figure 9.

The most interesting question is still, what happens to the particle–antiparticle pair production obviously necessary to explain the flat plateau in the single-particle inclusive y distribution. To some extent, the shapes of x, y or charge distributions for the four-body final state around 20 GeV/*c* resemble already those of inclusive distributions at much higher energies (see § 5). Thus, if there is central production at high energies, we may expect it to show up in four-body final states at about 20 GeV/*c*. This is allowed from exchange of quantum numbers (eg double pomeron) and is expected to contribute to the population close to the origin of figure 12. The cross section is not particularly large there, but there may be a 'ridge' between the two single-dissociation maxima in sectors A and B. The energy dependence of the matrix element in this central region has been studied by Yamdagni and Gavrilas (1971) in figure 19. It is still consistent with that of a mixture of diffraction-dissociation and exchange processes contributing there. The mass of the two-pion system is small in this region and therefore consistent with an s-wave assignment and the $(\pi^+\pi^-)$ to $(\pi^0\pi^0)$ ratio is not inconsistent with an $I = 0$ state, as would be required for double-pomeron exchange.

So central pair production cannot be ruled out completely, but the cross section seems to be very small at around 20 GeV/*c*. Does fragmentation of the incident particles first have to 'saturate' (eg in respect to four-momentum transfer) before central pair production is allowed, so that pionization would only be possible in high-multiplicity reactions? Exclusive distributions at 300 GeV/c or better at ISR energies may be able to tell.

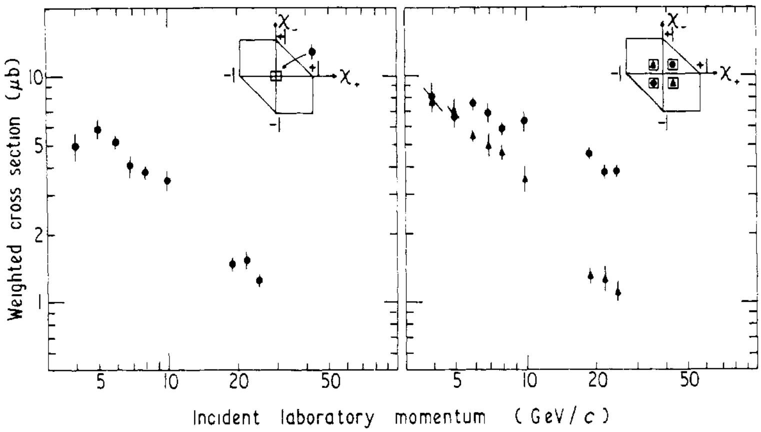

Figure 19. Weighted cross sections for pp → p$\pi^+\pi^-$p in the indicated LPS regions as functions of the incoming laboratory momentum (Yamdagni and Gavrilas 1971).

5. Correlations between exclusive distributions

We have looked at inclusive and exclusive reactions rather independently, so far. Of course they are closely connected: if we could study *all* multiplicities, we could write the inclusive cross section as a linear combination of all exclusive cross sections and vice versa. We cannot study all multiplicities, but from the few we can, it seems that the smoothness of inclusive cross sections may be due to some 'conspiracy' of exclusive ones.

For the integrated cross sections, this is shown in figure 20, where the total π^-p cross section and some of the contributions from exclusive channels are plotted against the incoming laboratory momentum (Honecker *et al* 1969, Hansen *et al* 1971). One could imagine that there might be a lot of structure in the total cross section due to rising and falling channel cross sections. Above 2 GeV/c, however, there is very little structure in the total cross section, so that there seems to exist some conspiracy between the (inelastic) exclusive cross sections to give the smooth energy dependence in their sum. As pointed out by Van Hove (1972), this smoothing effect is visible in a somewhat similar form in single-particle distributions.

Firstly, figures 21(a, b) give the x_s distributions of the 'slow' negative pion, produced in reaction (18), $\pi^-\text{p} \rightarrow \pi_f^-\pi_s^-\pi^+\text{p}$ at 16 GeV/c, for events (a) $x_+ > 0$ and (b) $x_+ < 0$, respectively. These distributions essentially correspond to the integration over X_+ in the right and left halves of figure 12(a), respectively, and are to be compared with the distribution integrated over all X_+ in figure 21(c). Besides demonstrating how much of the separation is lost by the full X_+ integration, this comparison shows that there is nothing left of any abrupt change in the sum, as for example, that at $x_s = 0$ in figures 21(a, b). This may, of course, change with increasing energy, when the two single-dissociation peaks get better separated.

Secondly, x distributions (now for all negative pions) are given in figure 22 for exclusive reactions of multiplicities between 3 and 8. Different multiplicities obviously

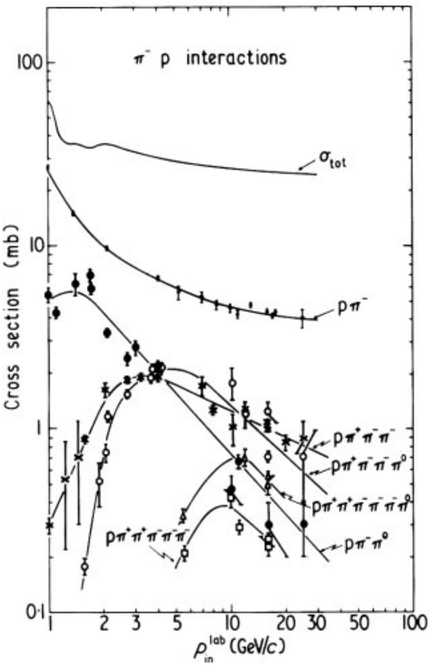

Figure 20. Variation of cross sections for exclusive reactions with incoming laboratory momentum, compared to that of the total and elastic cross sections (Honecker *et al* 1969).

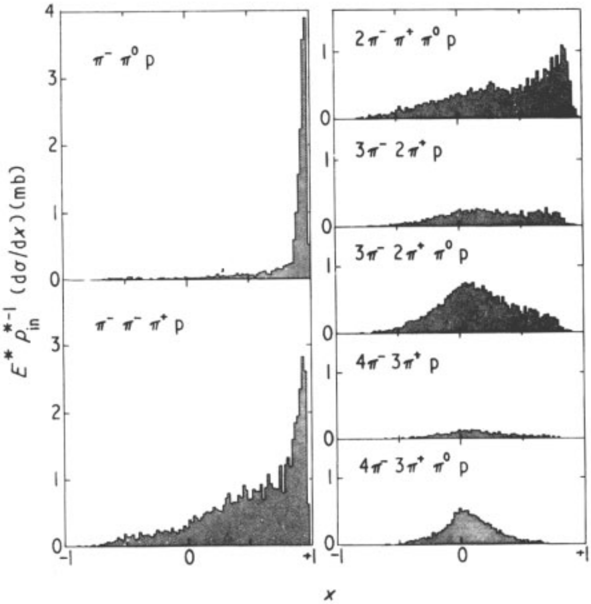

Figure 22. Single-particle x distribution for negative pions produced in exclusive π^-p reactions at 16 GeV/c (ABBCCHW collaboration 1972).

dominate in rather different x regions and vary largely in shape. Should, the x distribution of the four-body reaction ever separate according to figures 21(a, b) into two peaks, there will be high multiplicity channels contributing to the $x = 0$ region only, so that the separation will be lost in the sum. It is surprising, however, *how smooth* in x the sum actually becomes (see figure 5(a)). As shown in figure 23 for pp reactions at 19 GeV/c by Bøggild *et al* (1971) and Satz (1971), the two diffraction maxima in the four-body reaction start to be separated in y, but higher multiplicities contribute close to $y = 0$ mainly, to give the smooth full curve representing the inclusive distribution peaking at at $y = 0$.

A charge distribution like that for π^+p in figure 4(b) is decomposed into the individual channels by Łoskiewicz (1972) in figure 24. It is surprising how different the channel distributions look from the total and from each other. Again, they add up to a rather smooth total distribution.

So we have learned that a lot of structure in the exclusive distributions gets lost to give a smooth inclusive distribution. The question is whether this is an accidental averaging effect or some additional information obtainable from inclusive distributions only. Experimentally, it seems to be one of the main reasons to always study inclusive and exclusive distributions *simultaneously* on the *same* sample of data.

6. So what have we learned?

The answer to this question would be given differently by different authors and can be taken so far only as a basis for discussion. Such a discussion is interesting because it suggests further clarifying work. We try to summarize the answer here in the three

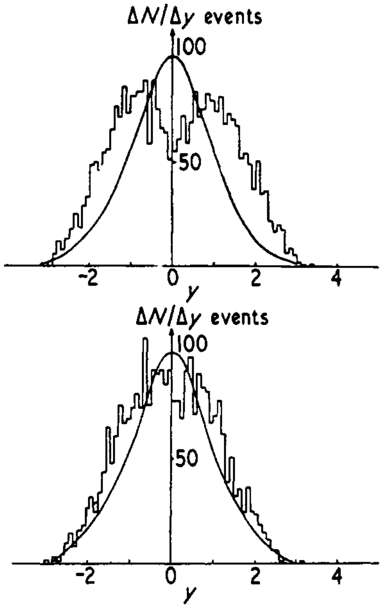

Figure 23. Comparison of the π^- distribution for pp $\rightarrow$ p$\pi^+\pi^-$p and pp $\rightarrow$ p$\pi^+\pi^+\pi^-$n at 19 GeV/c with the inclusive distribution at the same energy (Satz 1971).

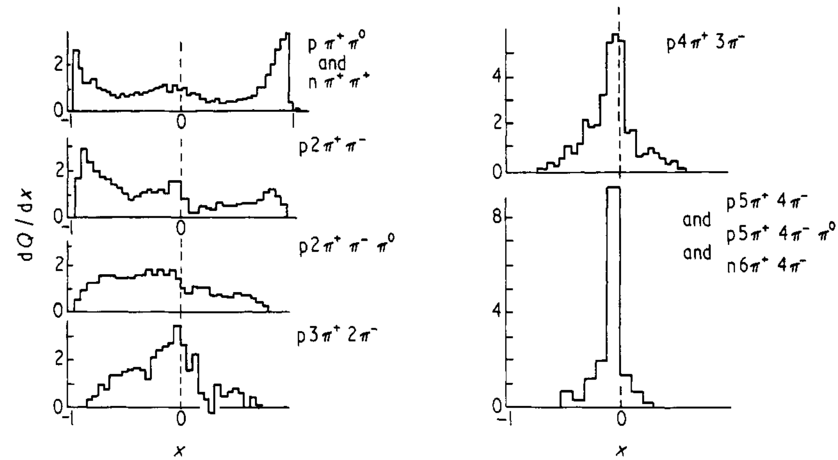

Figure 24. Charge distribution for exclusive π^+p reactions at 8 GeV/c (Łoskiewicz 1972).

steps: (i) do not be led by a special model, for the moment; (ii) be differential but still comprehensive; (iii) some physics.

We make the first point, because there is no real theory to guide us and data are extremely complex, certainly more complex than provided in any *a priori* model. There is, of course, nothing against building a model on the basis of a differential but still comprehensive data presentation and analysis.

One powerful way, especially with ISR and 400 GeV/c accelerators in mind, is an analysis in longitudinal phase space (allowing a study of transverse phase space in second approximation). Consistently performed inclusively and exclusively on the same data it is far more ambitious than an experimental result quickly obtained only to distinguish between two special models.

The systematic analysis has so far led to the observation of strong correlations, probably due to different regimes in particle production with small transverse momenta:

(i) *Diffraction dissociation or fragmentation* can be isolated as the dominant mechanism (above $p_{in}^{lab} \sim 5$ GeV/c) in low-multiplicity reactions. It contributes essentially to phase-space regions where $p_{\parallel}^{*} \gg p_{\perp}$, that is, to regions close to the phase-space boundary in the longitudinal variables, but at least at low energies, fragmentation of the projectile and that of the target overlap largely in inclusive single-particle distributions. Scaling and vertex factorization are well established for this mechanism and hold inclusively as well as exclusively. The purity of the production mechanism (essentially vacuum exchange) and the exclusive structure (quantum numbers, in particular isospin and partial waves) of the dissociating systems are fairly well known.

(ii) *Central particle production* ($p_{\parallel}^{*} \sim p_{\perp}$) is mainly due to high-multiplicity reactions. At 1500 GeV/c, this central region does not clearly show scaling and factorization yet, though an extrapolation to $s = \infty$ indicates them to eventually hold there. The internal structure of this regime is very little known, since the high dimensionality of phase space and the large overlap of many possible production mechanisms (including double-dissociation) limit the exclusive study so far only performed up to about 20 GeV/c. The fundamental question, whether this central region is due to more or less isotropic high-multiplicity reactions or to central pair production may be possible to decide from future *exclusive* studies of ISR and 400 GeV/c accelerator data.

(iii) *Nonvacuum exchange processes* are well studied as quasi-two-body reactions in low-multiplicity final states. At moderate energies ($\sim$20 GeV/c) they contribute to intermediate x regions filling in the 'valley' possibly separating regimes (i) and (ii) in two-particle distributions at high energies. Already suppressed in three- and four-particle final states at about 20 GeV/c, exchange processes may 'withdraw' to higher multiplicities with increasing energy.

When Pythagoras had his famous idea about the sides of the rectangular triangle, he sacrificed as many as 100 oxen to the gods from whom this idea came. We have not yet come so far that we can relax, write down the theory and sacrifice, but I believe we are at least starting to learn how to ask, systematically.

References

Aachen–Berlin–Bonn–CERN–Cracow–Heidelberg–Warsaw collaboration 1972 to be published

Antipov Yu M *et al* 1972 *Proc. 3rd Int. Conf. on Experimental Meson Spectroscopy, Philadelphia*

Ascoli G *et al* 1970 *Phys. Rev. Lett.* **25** 962

Beaupré J V *et al* 1971 *Nucl. Phys.* B **35** 61

—— 1972a *Nucl. Phys.* B **46** 1

—— 1972b *Phys. Lett.* **41B** 393

Benecke J, Chou T T, Yang C N and Yen F 1969 *Phys. Rev.* **188** 2159

Bertin A *et al* 1972 *Phys. Lett.* **38B** 260

Bialas A *et al* 1969 *Nucl. Phys.* B **11** 479

Bøggild H *et al* 1971 *Proc. Int. Conf. on Elementary Particles, Amsterdam* 1971 ed A G Tenner and M J G Veltman (Amsterdam)

Brandenburg G *et al* 1972 *Nucl. Phys.* B **45** 397

Brandt S, Peyrou Ch, Sosnowski R and Wroblewski A 1964 *Phys. Lett.* **12** 57

Brau J E *et al* 1971 *Phys. Rev. Lett.* **27** 1481

Breidenbach B, Flügge G, Schubert K R and Williams E G H 1972 *CERN Report* CERN/NP 72/6

Chan Hong-Mo, Hsue C S, Quigg C and Jiunn-Ming Wang 1971 *Phys. Rev. Lett.* **26** 672

Denisov S P *et al* 1971 *Phys. Lett.* **36B** 415, 528

Deutschmann M *et al* 1972a *Nucl. Phys.* B **50** 61

—— 1972b *Nucl. Phys.* B **50** 80

De Wolf E, Goldschmidt-Clermont Y, Grard F and Verbeure F 1972 *Nucl. Phys.* B **46** 333

Ferbel T 1972 *Phys. Rev. Lett.* **29** 448

Feynman R P 1969 *Phys. Rev. Lett.* **23** 1415

Giacomelli G 1972 *Proc. 16th Int. Conf. on High Energy Physics, Batavia* 1972

Hansen J D, Kittel W and Morrison D R O 1971 *Nucl. Phys.* B **25** 605

Honecker R *et al* 1969 *Nucl. Phys.* B **13** 571

Idschok U *et al* 1972 *Proc. 16th Int. Conf. on High Energy Physics, Batavia* 1972

Kittel W, Ratti S and Van Hove L 1971 *Nucl. Phys.* B **30** 333

Koba Z 1972 *Proc. 3rd Int. Colloq. on Multiparticle Reactions, Zakopane* 1972 ed A. Bialas *et al* (Cracow and Warsaw) p 314

Łoskiewicz J 1972 *Proc. 3rd Int. Colloq. on Multiparticle Reactions, Zakopane* 1972 ed A Bialas *et al* (Cracow and Warsaw) p 147

Meyer H and Struczinski W 1972 *DESY Report* DESY 72/40

Moffeit K C *et al* 1972 *Phys. Rev.* D **5** 1603

Morrison D R O 1972a *Proc. 4th Int. Conf. on High Energy Collisions, Oxford* 1972 ed J R Smith (Oxford) p 253

—— 1972b *Proc. 3rd Int. Colloq. on Multiparticle Reactions, Zakopane* 1972 ed A Bialas *et al* (Cracow and Warsaw) p 348

Mück H J *et al* 1972 *Proc. 4th Int. Conf. on High Energy Collisions, Oxford* 1972

Otter G 1972 *Proc. 12th Cracow School of Theoretical Physics, Zakopane* 1972

Rushbrooke J 1972 *Proc. 3rd Int. Colloq. on Multiparticle Reactions, Zakopane* 1972 ed A Bialas *et al* (Cracow and Warsaw) p 442

Satz H 1971 *Proc. Int. Conf. on Elementary Particles, Amsterdam* 1971 ed A G Tenner and M J G Veltman (Amsterdam) p 39

Sens J C 1972 *Proc. 4th Int. Conf. on High Energy Collisions, Oxford* 1972 ed J R Smith (Oxford) p 177

Van Hove L 1969a *Phys. Lett.* **28B** 429

—— 1969b *Nucl. Phys.* B **9** 331

—— 1971 *Phys. Rep.* **1C** 347

—— 1972 *Lectures at the International School of Subnuclear Physics, Erice* 1972

Verbeure F 1972a *Proc. 3rd Int. Colloq. on Multiparticle Reactions, Zakopane* 1972 ed A Bialas *et al* (Cracow and Warsaw) p 2

—— 1972b *CERN Report* CERN/DPhII/Phys 72-42

Yamdagni N K and Ljung S 1971 *Phys. Lett.* **37B** 117

Yamdagni N K and Gavrilas M 1971 *Proc. Int. Conf. on Elementary Particles, Amsterdam* 1971 ed A G Tenner and M J G Veltman (Amsterdam)